

Dynamic simulation of one-stage Oxford split-Stirling cryocooler and comparison with experiment

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Received 27 March 2002; accepted 4 July 2002

Abstract

In this paper, a mathematical model of a one-stage Oxford split-Stirling cryocooler has been made to simulate its dynamic performance. A set of governing equations has been solved by a numerical method. The present paper emphasizes the validation of this model against experimental results for the Oxford split-Stirling cryocooler. The comparison shows acceptable agreement between the simulation and experimental results. The simulated results on compression work have relative discrepancies of 6.8% (cold-tip temperature 80 K) and 6.5% (cold-tip temperature 54 K) with their experimental results respectively. The greatest deviations for the minimum pressure and net cooling power are about 6.0% and 8.0% respectively.

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Keywords: Oxford split-Stirling cryocooler; Refrigeration; Numerical simulation

1. Introduction

Oxford split-Stirling cryocoolers are having been developed since its invention by the researchers in Oxford University at the end of 1970s [1], and many special components or technologies such as a linear compressor, no-contact seals, flexure springs, controlling contaminants technologies and etc. have been introduced during the manufacturing and assembling processes. At present, because of their maturity and high reliability, Oxford split-Stirling cryocoolers have become quite popular in many fields, space applications in particular [4,5].

In an Oxford Stirling cryocooler, the regenerator always contains a long stack of phosphor bronze gauze discs. The compression piston and the displacer move at a high frequency (30–60 Hz), and the working gas's oscillating movement is very complicated [3,6,9]. All these make it very difficult for the researchers to grasp the genuine performance characteristics, and rely on a lot of experiments or experimental data during the thermal

design process. In order to reduce the development period of an Oxford split-Stirling cryocooler, a dynamic simulation program has been developed and a visualized experimental project has been put forward by us.

Since Finkelstein's pioneering the nodal network analysis of Stirling Engine [2], a number of nodal analysis systems are available for the small Stirling cryocoolers in the literature [7–9], however, none of them has been validated extensively by experiments. To the best of the author's knowledge, a mathematical model for a one-stage Oxford split-Stirling cryocooler has been described in this paper. A set of differential equations based on the conservation of mass, energy, and momentum for working gas or matrix are solved by a numerical method. The present paper emphasizes the validation of the above model against experimental results for an Oxford split-Stirling cryocooler and acceptable agreement was found between the two.

2. Physical model and governing equations

The physical model for an Oxford split-Stirling cryocooler is shown in Fig. 1. In order to set up a comparatively simplified and practical mathematic model, the basic assumptions are made as follows:

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Nomenclature

A	heat transfer area
A_H	heat transfer area for hot chamber
c_p	gas specific heat
c_m	matrix specific heat
D_h	equivalent diameter
f	friction factor or frequency
f_1, f_2	friction factor
I	electric current
m	gas mass
$\dot{m}$	mass flow rate
p	pressure
Pr	Prantle's number
Q	heat or cooling power
R	gas constant or electric resistance of winding coil
Re	Reynold's number
S	stroke
T	temperature or period
u	gas flow velocity
V	volume
$\dot{W}$	work rate
x	coordinate of flow direction

Greek letters

α	heat transfer coefficient
ε	efficiency or emissivity
Φ	differential phase angle
λ	thermal conductivity
μ	dynamic viscosity or modified empirical coefficient
ρ	density

σ	convergent constant
τ	time
γ	ratio of specific heat
ψ	modified resistance coefficient

Subscripts

A	ambient
C	compression chamber or compressor
conduction	conduction loss
cp	compressor
D	displacer
E	cold chamber
effective	effective value
emit	radiation loss
f	boundary of control volume
gross	gross cooling power
h	hydraulic
H	hot chamber or hot end
i	the i th control volume
input	compressor input power
j	the j th control volume
m	matrix
max	maximum
net	net cooling power
out	outlet
pump	pump loss
R	regenerator
reg	regenerator or regenerative loss
shuttle	shuttle loss
t	tube or hose
W	wall

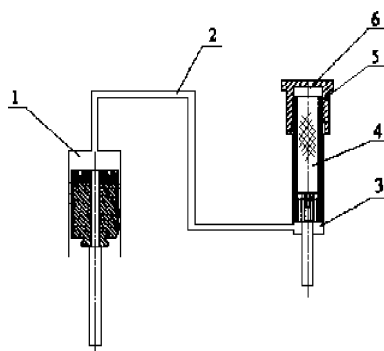


Fig. 1. Physical model: (1) compression chamber, (2) connecting hose, (3) hot chamber, (4) regenerator, (5) displacer, (6) cold chamber.

1. one-dimensional (1D) flow and heat transfer;
2. ideal gas and constant properties;
3. negligible axial conduction and linear temperature distribution of the regenerator matrix;
4. negligible clearance leakage between the cylinder and piston, or the cylinder and displacer;

5. constant wall temperature of the hot and cold chambers;
6. momentum and other equations are not coupled.

The following describe dynamic model and governing equations for the five components: namely, compression chamber, connecting hose, hot chamber, regenerator, and cold chamber, respectively.

2.1. Adiabatic compressor model and governing equations

For the one-stage Oxford split-Stirling cryocooler, the pressure wave is provided with a linear compressor coupled to the hot chamber through a connecting hose, as shown in Fig. 1. This design can make the cold-tip not be affected greatly by vibration from the compressor. It is particularly suitable to coupling with the infrared focal plane in many applications.

Since the compression piston motion is very fast and the mixing in the compression chamber is severe, the gas temperature and pressure inside the compression

chamber are assumed to uniform. The compressor cylinder is assumed to be adiabatic in the analysis, since each of the compression and expansion processes occurs in such a short period of time that little heat exchange between the gas and the cylinder wall could be effectuated. The gas adiabatically compressed in the cylinder is assumed to be cooled to room temperature by the cylinder lid and compressor base frame, which were made of copper and industry titanium respectively. The cylinder lid and compressor base frame is regarded as the after-cooler and assumed to be perfectly effective, so that the temperature of the gas leaving it is always equal to its wall temperature.

Applying the first law of thermodynamics to the control volume drawn around the volume swept by the compression piston in the cylinder, the following equations can be obtained for ideal gas:

$$dV_C = \frac{1}{p_C} \left(RT_a dm_C - \frac{V_C}{\gamma} dp_C \right) \quad (dm_C > 0) \quad (1)$$

$$dV_C = V_C \left(\frac{dm_C}{m_C} - \gamma \frac{dp_C}{p_C} \right) \quad (dm_C < 0) \quad (2)$$

where $\dot{m}_C$, V_C and p_C are, respectively, the mass, volume, and pressure of the gas in the control volume; T_a is the temperature of copper cylinder lid and titanium compressor base frame, which is assumed to be same as room temperature; and γ is the ratio of specific heats. The increment of mass in the above two equations, dm_C , is related to the mass flow rate into or out of the cylinder, $\dot{m}_C$. If $\dot{m}_C$ is defined as positive when the gas is leaving the cylinder, the conservation of mass gives

$$dm_C = -\dot{m}_C d\tau \quad (3)$$

For the Oxford split-Stirling cryocooler, the compression piston is driven by a linear motor. The equation of volume variation for the compression chamber can be written as follow:

$$V_C = \bar{V}_C + \Delta V_C \sin(2\pi f\tau) \quad (4)$$

where $\bar{V}_C$ and ΔV_C are, respectively, the average volume, and volume variation amplitude of the gas in the control volume; and f is the frequency for the moving compression piston. In practice, volume variations for the compression chamber and cold chamber are not exactly sinusoidal wave; so their equations should be rectified according to their measured results.

State equation for the gas in the control volume:

$$p_C V_C = m_C R T_C \quad (5)$$

From the above five equations, the variation with time of p_C and $\dot{m}_C$, may be determined by numerical calculation. The compression power and input power of the compressor can be written as follows:

$$\dot{W}_{cp} = -f \oint p_C dV_C \quad (6)$$

$$\dot{W}_{input} = \dot{W}_{cp} + I^2 R \quad (7)$$

where I and R are the current and resistance for winding coil of the linear motor.

The compressor efficiency may be calculated by

$$\varepsilon_{cp} = \frac{\dot{W}_{cp}}{\dot{W}_{input}} \quad (8)$$

2.2. Connecting hose model and governing equations

The connecting hose links the compressor and the expander. The gas flow field is assumed to be 1D. For governing equations being simplified easily, the second order viscous and inertia terms in the momentum equation are neglected. However, the second order viscous friction is taking account by a modified resistance coefficient ψ which is calculated by piecewise linear approximation [10]. Therefore, the viscous resistance is proportional to the mass flow rate with a proportional constant ψ depending on the amplitude of the mass flow rate.

Since the connecting hose is small in diameter and thick in wall thickness in order to withstand the high gas pressure, the gas in the connecting hose is assumed to undergo an isothermal process with room temperature T_a . If we divide the connecting hose into I segments, the governing mass and momentum equations for the gas in the i th ($i = 1, 2, \dots, I$) segment connecting hose are given by

$$\frac{1}{RT_a} \frac{\partial (p_t)_i}{\partial \tau} + \frac{1}{A_t} \frac{\partial (\dot{m}_t)_i}{\partial x} = 0 \quad (9)$$

$$\frac{1}{A_t} \frac{\partial (\dot{m}_t)_i}{\partial \tau} + \frac{\partial (p_t)_i}{\partial x} + \frac{\psi}{A_t} (\dot{m}_t)_i = 0 \quad (10)$$

As for the modified resistance coefficient ψ , the following empirical formula for oscillating gas flow in long tube from Roach and Bell [11] was adopted.

$$\psi = 0.1556 [\bar{\rho}_t (u_t)_{\max} d_{th} / \mu_t]^{-0.201} [(u_t)_{\max} / d_{th}] \quad (11)$$

where $(u_t)_{\max}$ and d_{th} are the maximum velocity of the oscillating gas flows, and the hydraulic diameter of the connecting hose respectively. It is worth noting that ψ is considered to be a constant during the modeling, but should be adjusted by numerical iteration during the computation to give a correct value for the responding maximum gas velocity $(u_t)_{\max}$.

2.3. Hot chamber model and governing equations

Being similar to the compression chamber, the gas temperature and pressure are assumed uniform in the hot chamber since the mixing effect is severe. The mass and energy equations are given by

$$\frac{dm_H}{d\tau} = (\dot{m}_t)_I - (\dot{m}_t)_{in} \quad (12)$$

$$m_H c_p (\Delta T_H)_{\Delta\tau} = \alpha (A_H)_\tau ((T_H)_\tau - T_{HW}) \quad (13)$$

The volume variation for the hot chamber is in opposite phase with that for the cold chamber which was given by equation (21).

2.4. Regenerator model and governing equations

The regenerator is divided into many subsystems and each subsystem can exchange work, heat and mass with the surroundings through its boundaries. The parameters of each subsystem vary with the time and are solved with the lumped method. It is defined mass flow rate being positive if the gas flows from the hot chamber to the cold one and negative for the opposite direction. Since the temperature gradient is large within the regenerator, we divided the regenerator into many subsystems. For each subsystem, j ($j = 1, 2, \dots, J$), the governing equations can be written as follows. Continuity for gas flow:

$$\frac{\partial m}{\partial \tau} = \dot{m}_j - \dot{m}_{j+1} \quad (14)$$

Conservation of momentum for the gas (by simplification):

$$\frac{dp_j}{dx} = -f \frac{1}{D_h} \frac{\rho_j u_j^2}{2} \frac{u_j}{|u_j|} \quad (15)$$

Conservation of energy for the gas:

$$c_p \frac{d(m_j T_j)}{d\tau} + c_p (\dot{m} T)_{j+1} - c_p (\dot{m} T)_j - \alpha_j A_j (T_{Wj} - T_j) - V_j \frac{dp_j}{d\tau} = 0 \quad (16)$$

Conservation of energy for the matrix:

$$\rho_m c_m \frac{\partial T_{Wj}}{\partial \tau} + \alpha_j A_j (T_{Wj} - T_j) = 0 \quad (17)$$

State equation for the gas:

$$p_j = \rho_j R T_j \quad (18)$$

2.5. Cold chamber model and governing equations

Being similar to that in the hot chamber, a lumped model was applied to the gas in the cold chamber. The mass and energy equations are given by

$$\frac{dm_E}{d\tau} = (\dot{m}_R)_J \quad (19)$$

$$m_E c_p (\Delta T_E)_{\Delta\tau} = \alpha (A_E)_\tau ((T_E)_\tau - T_{EW}) \quad (20)$$

The displacer is driven by a linear motor, and the motion phase difference maintains constant Φ with the

compression piston motion. The equations of volume variation for the cold chamber can be written as follow:

$$V_E = \bar{V}_E + \Delta V_E \sin(2\pi f \tau + \Phi) \quad (21)$$

Being similar to Eq. (4), Eq. (21) should be rectified according to experimental results.

The gross cooling power of the cooler can be written as follow:

$$\dot{Q}_{gross} = f \oint p_E dV_E \quad (22)$$

We define $\dot{Q}_{conduction}$, $\dot{Q}_{reg}$, $\dot{Q}_{shuttle}$, $\dot{Q}_{pump}$, and $\dot{Q}_{emit}$ as solid conduction loss, regenerator inefficiency loss, shuttle loss, pump loss and radiation loss, respectively. Other losses of cooling, such as residual gas conduction loss, leakage loss, etc. are considered to be negligible. Therefore, the net cooling power of the cooler is

$$\dot{Q}_{net} = \dot{Q}_{gross} - \dot{Q}_{conduction} - \dot{Q}_{reg} - \dot{Q}_{shuttle} - \dot{Q}_{pump} - \dot{Q}_{emit} \quad (23)$$

3. Losses of cooling

In order to know the net cooling power available at the cold end of the cold finger, we need to calculate each loss term in Eq. (23). The regenerator inefficiency loss and solid conduction loss are the most important.

3.1. Regenerator inefficiency loss

In the regenerator of an Oxford Stirling cryocooler, the mass flow rate, temperature, and pressure are all functions of time. Oscillating flow occurs rather than steady state flow. There being considerable void volume in the regenerator, the fluctuating gas pressure gave rise to the variable mass in the regenerator, and differential mass flow rate at different section of the regenerator. In view of the fluctuating gas temperature and variable mass flow rate, enthalpy flow from hot end to cold end of the regenerator, that is to say, regenerative inefficiency loss can exist. This loss term can be calculated through numerical method by

$$\dot{Q}_{reg} = f \oint (\dot{m}_R)_{out} c_p T(L_R, \tau) d\tau \quad (24)$$

3.2. Solid conduction loss

The solid conduction loss comprises the heat conductions through the expander cylinder, and the regenerator, including the wall (displacer) and the matrix, which can be calculated by

$$(\dot{Q}_{\text{conduction}})_{\text{R}} = \mu \lambda_{\text{effective}} (\pi D_{\text{R}}^2 / 4) \frac{T_{\text{RH}} - T_{\text{RL}}}{L_{\text{R}}} \quad (25)$$

where $\lambda_{\text{effective}}$ is the effective thermal conductivity to the regenerator; and μ is a modified empirical coefficient, which takes the matrix material, matrix thermal conductivity, matrix stacking, etc. into account.

Owing to large temperature gradient on the regenerator and displacer, the matrix thermal conductivity may be varied to a great extent. Therefore, the effective thermal conductivity to the regenerator is given by

$$\lambda_{\text{effective}} = \frac{1}{T_{\text{RH}} - T_{\text{RE}}} \int_{T_{\text{RE}}}^{T_{\text{RH}}} \lambda(T) dT \quad (26)$$

The heat conduction loss caused by the cold-finger cylinder wall can be calculated by

$$(\dot{Q}_{\text{conduction}})_{\text{cylinder}} = \frac{A}{L} \int_{T_{\text{EW}}}^{T_{\text{HW}}} \lambda(T) dT \quad (27)$$

So the total solid conduction loss is

$$\dot{Q}_{\text{conduction}} = (\dot{Q}_{\text{conduction}})_{\text{R}} + (\dot{Q}_{\text{conduction}})_{\text{cylinder}} \quad (28)$$

3.3. Shuttle loss

The displacer/regenerator shuttle heat loss is evaluated from the relation of Baik and Chang [12]:

$$\dot{Q}_{\text{shuttle}} = \frac{\pi D_{\text{D}} S_{\text{D}}}{4} 2\pi f \lambda_{\text{D}} \rho_{\text{D}} (c_{\text{p}})_{\text{D}} T_{10} \sin \phi \quad (29)$$

where

$$T_{10} = \Delta T_{\text{D}} S_{\text{D}} \frac{a_2}{\sqrt{\left(\frac{a_1+a_2}{\sqrt{2}}\right)^2 + \left(\frac{a_1+a_2}{\sqrt{2}} + a_1 a_2\right)^2}} \quad (30)$$

$$\phi = \tan^{-1} \left(\frac{(a_1 + a_2)/\sqrt{2} + a_1 a_2}{(a_1 + a_2)/\sqrt{2}} \right) \quad (31)$$

$$a_1 = \frac{\lambda_{\text{D}}}{h} \sqrt{\frac{2\pi f}{\alpha_{\text{D}}}}, \quad a_2 = \frac{\lambda_{\text{W}}}{h} \sqrt{\frac{2\pi f}{\alpha_{\text{W}}}} \quad (32)$$

where D_{D} is the displacer outside diameter; S_{D} is the displacer stroke; λ_{D} is the displacer thermal conductivity; λ_{W} is the cylinder wall thermal conductivity; ρ_{D} is the displacer density; $(c_{\text{p}})_{\text{D}}$ is the displacer heat capacity; ΔT_{D} is the axial temperature gradient of the displacer; α_{D} and α_{W} are the thermal diffusivity of the displacer and the cylinder wall; h is the heat transfer coefficient.

3.4. Pump loss

Owing to the clearance between the displacer and the cold-finger cylinder, some gas is moving periodically between the clearance and the cold chamber, which

can give rise to cooling loss termed as pump loss. The pump loss can be calculated by the following equation [13]:

$$\dot{Q}_{\text{shuttle}} = 4.04 L_{\text{D}} (T_{\text{H}} - T_{\text{E}}) \delta^{2.6} \left(\frac{\pi D_{\text{D}}}{\lambda} \right)^{0.6} \times \left[\frac{2f c_{\text{p}}}{(T_{\text{H}} + T_{\text{E}})} (\Delta p)_{\text{peak-peak}} \right]^{1.6} \quad (33)$$

3.5. Radiation loss

There being large temperature difference between the cold-tip and the surroundings with constant ambient temperature, radiation loss of cooling power may occur. When the coolers works at stable state, the cold-tip temperature T_{EW} is almost constant with a very small fluctuation in practice. The radiation loss can be calculated by

$$\dot{Q}_{\text{emit}} = 5.77 \times 10^{-4} \varepsilon A_{\text{E}} \left[\left(\frac{T_{\text{HW}}}{100} \right)^4 - \left(\frac{T_{\text{EW}}}{100} \right)^4 \right] \quad (34)$$

where

$$\varepsilon = \frac{1}{\frac{1}{\varepsilon_{\text{EW}}} + \frac{A_{\text{EW}}}{A_{\text{HW}}} \left(\frac{1}{\varepsilon_{\text{HW}}} - 1 \right)} \quad (35)$$

where ε_{EW} and ε_{HW} are the emissivities for the cold and hot surface respectively; A_{EW} and A_{HW} are the area for the cold and hot surface respectively.

4. Friction factor and heat transfer coefficients

Since we could not find the friction factor and heat transfer coefficient for the oscillating gas flow, and the transition from laminar to turbulent flow is not obvious in the Stirling or pulse tube cryocoolers, we adopted the steady friction factor and heat transfer coefficient from Ref. [14]. The friction factor is

$$f = \max(f_1, f_2) \quad (36)$$

where the function max is defined as the largest among the variables, that is to say, if f_1 is equal or larger than f_2 , $f = f_1$; on the contrary, $f = f_2$. f_1 and f_2 can be calculated by the following formulae respectively

$$f_1 = 1.6 + 180/Re \quad (37)$$

$$f_2 = 0.257 Re^{-0.157} \quad (38)$$

The heat transfer coefficient is calculated by

$$\alpha = \max(\alpha_1, \alpha_2) \quad (39)$$

where

$$\alpha_1 = 0.023 \lambda Re^{0.8} Pr^{0.4} / D_h \quad (40)$$

$$\alpha_2 = 1.471 \lambda Re^{0.536} Pr^{0.333} / D_h \quad (41)$$

5. Numerical method

The above set of differential equations governing gas flow in connecting hose or regenerator is non-linear and unsteady. Some efforts have been made to increase both the calculation accuracy and speed. A uniform network for space discretization and an implicit scheme for time are utilized during the solving process. The control volume boundary temperature T_{fi} is given by the second-order upwind difference scheme:

$$T_{fi} = 1.5T_{i-1} - 0.5T_{i-2} \quad (\dot{m} > 0) \quad (42)$$

$$T_{fi} = 1.5T_{i+1} - 0.5T_{i+2} \quad (\dot{m} < 0) \quad (43)$$

The discretization energy equation can be solved by the TDMA (tridiagonal matrix algorithm) method. The iteration results should be satisfied by periodic stable conditions for all parameters and the energy balance condition, i.e. the net heat transfer for the matrix of regenerator tends to zero during a cycle. The convergent equation can be written:

$$\oint_{\tau=T} (\delta Q)_m = \sigma \Rightarrow 0 \quad (44)$$

σ can be adjusted according the solution accuracy and computational speed.

6. Computational results and verification by experiments

Verification of our numerical modeling was carried out by comparing the simulation results to experimental data. The tested machine is a one-stage Oxford split-Stirling cryocooler constructed at Cryogenic Laboratory in Shanghai Institute of Technical Physics (SITP). The main parameters are as follows: compressor displacement 1.6 cm³; compression piston stroke 8 mm; displacer stroke 4 mm; frequency 40 Hz; initial charge pressure 1.3 MPa. The ambient temperature is 303 K. Dynamic pressures in the compression and hot chambers were measured with two piezoelectric transducers and charge amplifiers. The displacements of the compression piston and the displacer were measured with two linear variable-differential position transformers, enabling the volume change to be determined according to the driven mechanism respectively. The cold-tip temperature was measured with a platinum resistance thermometer.

Fig. 2 shows the volume variations versus time for the compression and cold chambers. Their measured results

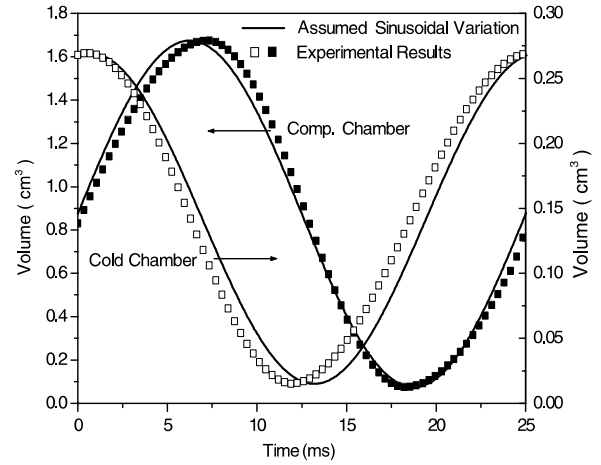


Fig. 2. Variation of volume with time, $\Phi = 78^\circ$.

indicate that time for the compression process is as about 1.2 times long as that for expansion process in some sort; these results deviate the assumption of the sinusoidal volume variations for the compression and cold chambers. So we utilized equations for the fitted curve to experimental results in place of Eqs. (4) and (21), respectively. According our experimental results while cold-tip temperature maintaining 80 K, the rectified displacement equations can be written by

$$V_C = \begin{cases} \bar{V}_C + \Delta V_C \sin(1.84\pi f \tau) & (\dot{x}_C \geq 0) \\ \bar{V}_C + \Delta V_C \sin(2.16\pi f \tau) & (\dot{x}_C < 0) \end{cases} \quad (45)$$

$$V_E = \begin{cases} \bar{V}_E + \Delta V_E \sin(1.84\pi f \tau + 0.92\Phi) & (\dot{x}_D \geq 0) \\ \bar{V}_E + \Delta V_E \sin(2.16\pi f \tau + 1.08\Phi) & (\dot{x}_D < 0) \end{cases} \quad (46)$$

Fig. 3 shows pressure variations versus time for the three chambers when the cold-tip temperature maintains at 80 K (Fig. 3A) and 54.3 K (Fig. 3B) respectively. The simulated dynamic pressures for gas in compression chamber, which were derived from Eqs. (4), (21) and Eqs. (45), (46) respectively, show that simulated result by utilizing rectified displacement Eqs. (45) and (46) has higher accuracy to the corresponding experimental result. In Fig. 3A, the tested pressure amplitudes for the compression and hot chambers decrease by 10.8% and 18.0% of their simulating results respectively; the tested pressure amplitude for hot chamber decrease by 12.0% of the tested result for the compression chamber. Comparing with the simulating result for the compression chamber, the simulating results for the hot and cold chambers lag 0.3 ms (4.3°) and 0.78 ms (11.2°) in phase respectively. The tested results are in phase with the simulating results for the compression and hot chambers. The tested maximum pressures for the compression and hot chambers are in consistent with the simulating ones, however, the measured minimum pressures increasing by about 4.8% and 6.0% than their simulating

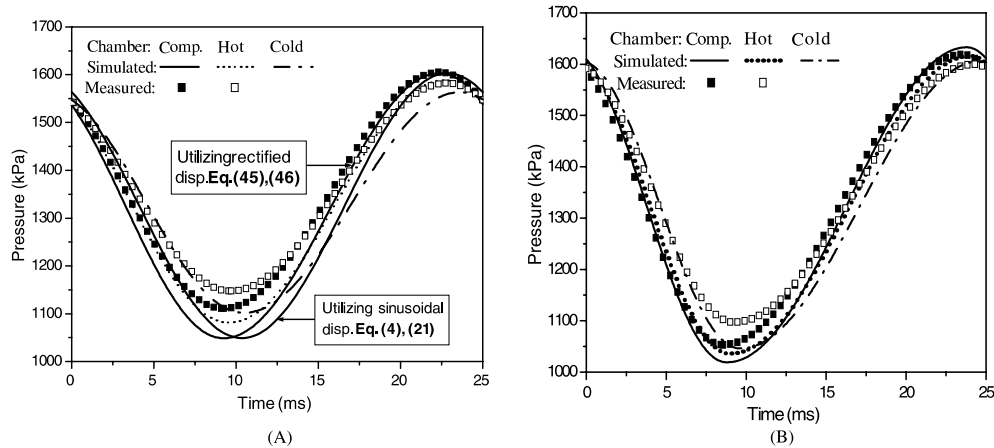


Fig. 3. Variation of pressure with time, $\Phi = 78^\circ$. (A) Cold-tip temperature 80 K, (B) cold-tip temperature 54 K.

results respectively. All these pressure amplitudes attenuation and phase delay occur analogously in Fig. 3B which the cold-tip temperature maintaining 54 K, however, the corresponding values are reduced because of the increased compressor pressure fluctuation (the simulated (measured) pressure amplitudes are 307 (283) and 277 (247) kPa for cold-tip temperature 54 and 80 K, respectively), and pressure ratio (the simulated (measured) pressure ratios are 1.60 (1.54) and 1.46 (1.38) for cold-tip temperature 54 and 80 K, respectively). Besides all above, Fig. 3A and B indicate that time of compressing process for cold-tip temperature 54 K is longer than that for cold-tip temperature 80 K in a working cycle. This phenomenon was caused by more load force and compressor piston providing with more work on gas for cold-tip temperature 54 K than 80 K. By comparing Figs. 2 and 3, we can deduce that with the cold-tip temperature lowering down, the phase differential angle between the volume and pressure variations for gas in the cold chamber increases from about 10.2 ms (147°) at 80 K of cold-tip temperature to 11.5 ms (165°) or so at 54 K of cold-tip temperature; this increasing phase differential angle was the main cause which gave rise to the net cooling power rapidly lowering from 630 mW at 80 K to no net cooling power at 54 K of cold-tip temperature. The above phase lag and amplitude reduction may caused by the damp effect of the connecting hose, our mathematical model not covering all information about the machine structure and movements, and the increased dead volume by instillation of transducers.

Figs. 4 and 5 show the simulated dynamic characteristics of the one-stage Oxford split-Stirling cryocooler as a function of run time, including instantaneous pressures in three chambers, mass flow rate at the compressor outlet, and the hot and cold ends of regenerator, and gas temperatures in the compression and cold chambers. Fig. 4 shows the mass flow rates at the hot and cold ends of regenerator are basically in phase,

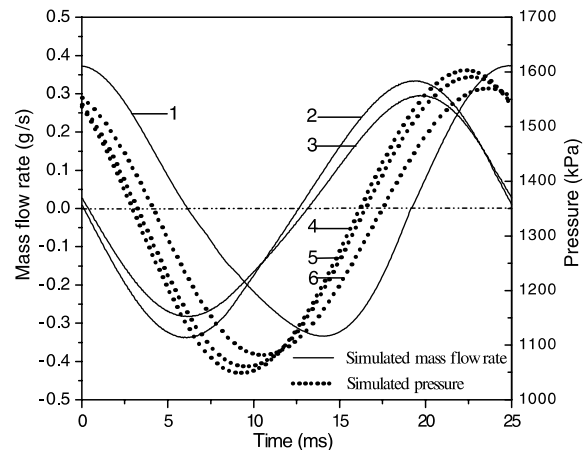


Fig. 4. Variation of mass flow rate and dynamic pressure versus time $\Phi = 78^\circ$, cold-tip temperature 80 K. 1, 5: outlet of compressor; 2, 4: hot end of regenerator; 3, 6: cold end of regenerator.

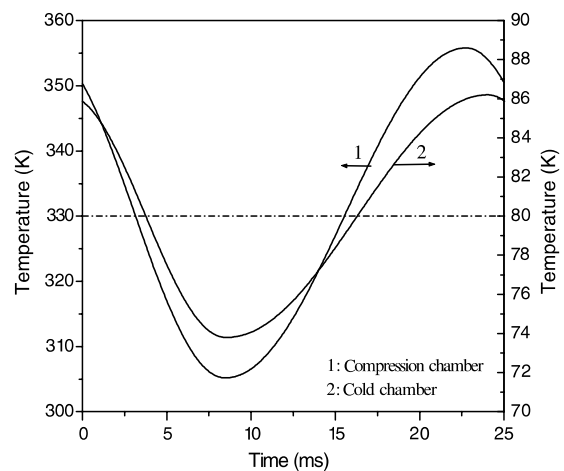


Fig. 5. Gas temperature at compression and cold chambers $\Phi = 78^\circ$, cold-tip temperature 80 K.

and are about 5.57 ms (80.2°) ahead of that at compressor outlet. This is in good agreement with the phase differential angle 80° between the compression piston and displacer movements. Fig. 5 is a plot of the gas temperatures at the compression and cold chambers at 80 K of cold-tip temperature. These two temperature waves are similarly in phase with their pressure waves respectively. The compression gas temperature oscillates between 305 and 355 K. For 630 mW cooling load at 80 K of the cold-tip temperature, the cold gas temperature fluctuates between 74 and 86 K.

The dynamic characteristics, including energy relations, instantaneous pressure, volume variations in the three chambers, and P – V diagrams, give a good understanding of the internal process and performance about an Oxford split-Stirling cryocooler. Pressure versus volume, or P – V plots are very useful for designing Stirling cryocoolers.

Fig. 6 shows comparison of the converged compressor P – V curve with the experimental data. The maximum pressure predicted agrees well with the measured results; however, the minimum pressure is about 4.8% (cold-tip temperature 80 K) and 2.3% (cold-tip temperature 54 K) lower. The shapes of both curves are similar, and the areas inside the predicted curves are about 17.2 W (cold-tip temperature 80 K) and 19.7 W (cold-tip temperature 54 K) respectively, comparing with that inside the measured curves being about 16.1 W (cold-tip temperature 80 K) and 18.5 W (cold-tip temperature 54 K), with relative discrepancies of 6.8% (cold-tip temperature 80 K) and 6.5% respectively. The input power of this compressor is approximately 22.1 W with a compressor efficiency $(W_{\text{input}} - I^2 R) / W_{\text{input}}$ of 78% while cold-tip temperature maintaining 80 K; and the input power is about 26.8 W with efficiency of 74% while cold-tip temperature maintaining 54 K. For cold-tip rated

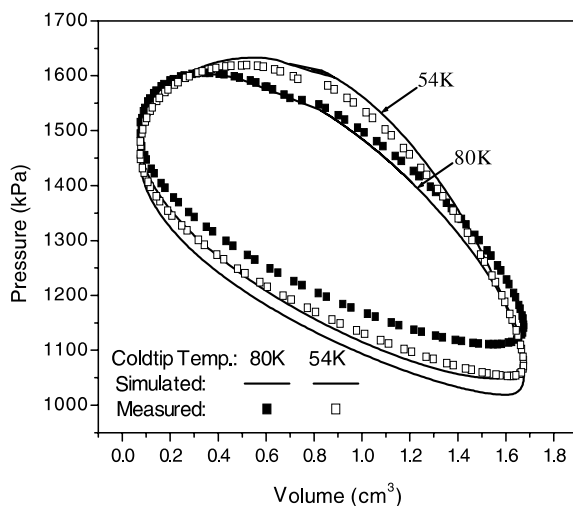


Fig. 6. Comparison of simulated P – V diagram against experimental results, $\Phi = 78^\circ$.

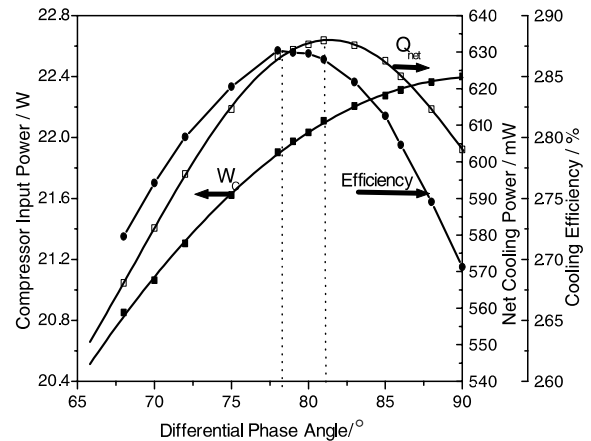


Fig. 7. Cooler performance versus differential phase angle between the displacer and compression piston.

working temperature of 80 K, the total consuming power of this tested unit is nearly 38.6 W, including approximate 4.5 W for the displacer driven motor and about 12 W for the controlling devices.

Fig. 7 shows the cooler performance versus differential angle between the displacer and compression piston. The compressor input power increases with the increase of the phase angle; however, there is an optimized phase angle of 78° for the cooling efficiency or 81° for the net cooling power respectively. We select operating phase angle of 80° in order to harmonize the cooling efficiency and net cooling power.

We simulated the operating cooling performances at different cold-tip temperature respectively, as were shown in Fig. 8. Fig. 8 indicates that the net cooling power is in approximately linear relation with the cold-tip temperature. The simulating results are congruent with the experimental ones when the cold end temperature is lower than that of liquid nitrogen (77 K); the simulating results increasingly deviate the experimental ones with increasing cold end temperature in the range

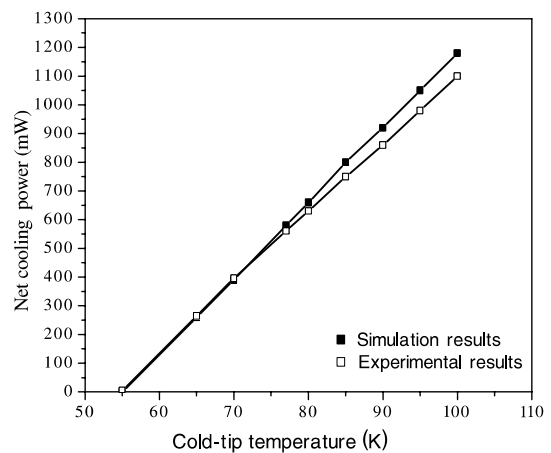


Fig. 8. Net cooling versus cold-tip temperature, $\Phi = 78^\circ$.

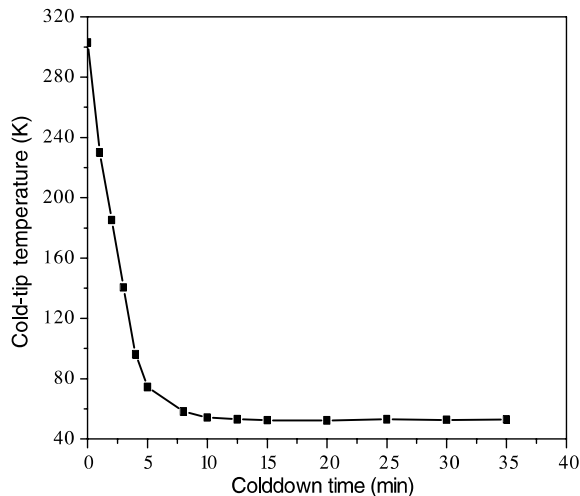


Fig. 9. Cold-tip temperature versus cooling time, $\Phi = 78^\circ$.

of 80–100 K. When the cold end temperature is 100 K, the simulating result is approximately 8.0% larger than the experimental one. Fig. 9 shows the cooling process for the Oxford split-Stirling cryocooler. This tested machine needs about 4.5 min for attaining 80 K with 630 mW cooling power, and about 10 min for attaining limit cooling temperature of approximately 54 K with no cooling load respectively.

7. Conclusion

The dynamic modeling for a one-stage Oxford split-Stirling cryocooler that has been developed can predict dynamic process and performances for which the structure dimensions and operating condition are given. Acceptable agreement between experiment and simulation has been found. The developed computer program can be utilized to develop an understanding of the in-

ternal process, and predicting the operating performances and dynamic characteristics.

Acknowledgements

The authors wish to thank Dr. L.W. Yang and Professor D. Chen for their valuable discussions and encouragements.

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